Math 10B with Professor Stankova
Quiz 11; Tuesday, 4/16/2019
Section #203; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False The 90% confidence interval will be smaller than the 95% confidence interval.

> **Solution:** With a smaller interval, we are less confident the true parameter will lie inside the range.

2. **TRUE** False The maximum likelihood estimate for p of a geometric distribution is the same one you get by setting $\hat{\mu} = \frac{1-\hat{p}}{\hat{p}}$.

> **Solution:** We showed this in discussion.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) The number of threes made during an NBA game is Poisson distributed. Last Saturday, the number of threes made were $14, 26, 25$, and 13. What is the 95% confidence interval for the number of 3's made during a game?

> **Solution:** We have
> $$\hat{\mu} = \bar{x} = \frac{14 + 26 + 25 + 13}{4} = \frac{78}{4} = 19.5.$$
> Then because this is a Poisson distribution, we have $\hat{\mu} = \hat{\lambda}$ so $\hat{\lambda} = 19.5$ and $\hat{\sigma} = \sqrt{\hat{\lambda}} = \sqrt{19.5}$. So the 95% confidence interval is
> $$(19.5 - \frac{2\sqrt{19.5}}{\sqrt{4}}, 19.5 + \frac{2\sqrt{19.5}}{\sqrt{4}}) = (19.5 - \sqrt{19.5}, 19.5 + \sqrt{19.5}).$$

(b) (5 points) Using the numbers from the previous part, calculate the maximum likelihood estimate for the parameter λ.

Solution: The likelihood function is $L(\lambda|14,26,25,13) = P(14,26,25,13|\lambda) = \frac{\lambda^{14}e^{-\lambda}}{14!} \cdot \frac{\lambda^{26}e^{-\lambda}}{26!} \cdot \frac{\lambda^{25}e^{-\lambda}}{25!} \cdot \frac{\lambda^{13}e^{-\lambda}}{13!} = \frac{\lambda^{78}e^{-4\lambda}}{14!26!25!13!}$. Taking the derivative and setting it equal to 0 gives

$$\lambda^{78}(-4e^{-4\lambda}) + 78\lambda^{77}e^{-4\lambda} = \lambda^{77}e^{-4\lambda}(-4\lambda + 78) = 0.$$

Thus $\lambda = 0$ or $\lambda = \frac{78}{4} = 19.5$. Since $\lambda = 0$ doesn't make sense and 19.5 is a local maximum, we have $\lambda = 19.5$.